

**STRUCTURAL MEMBRANE SHELLS  
EDGE CONDITIONS, IRREGULAR SHAPES, CONSTRUCTION  
AND DESIGN EXAMPLES**

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**ABSTRACT**

This paper extends the structural membrane approach to deal with specific edge conditions and irregularly arranged columns. Several design examples are given. Various possible construction methods are shown with special attention given to a combined thin shell roof projected to span a field house, pool, gymnasium and administration area.

**1. INTRODUCTION**

In earlier papers the basic structural membrane (SM) method has been outlined, utilizing domes (D) hyperbolic paraboloids (HP) and funnels (F) in the analysis of thin shells. This approach was limited to interior conditions of regular shells, assuming full continuity and symmetry at all edges. In the following presentation free edges, as well as irregular shapes, are discussed.

**2. FREE EDGE**

Initially, the structural membrane analysis assumes full symmetry along all edges. When the analysis is completed, all edges are made stress free by the introduction of stresses equal and opposite to the original ones. If these stresses were introduced uniformly along the edges, then no internal stress changes would take place. Instead, these forces are carried as concentrated loads to the thrust lines as described in an earlier companion paper. The resulting moments and shears along the edges are handled separately within the shell, assuming the shell area to work as deep beams. Proper consideration must be given to the effect of any curved edge outlines.

**3. IRREGULAR SHAPES**

Any structural membrane layout can be divided into a series of triangular areas, which are referred to as the SM Base Elements (SMBE). The corners of the triangular areas coincide with the centers of the domes, the HPs and the funnels. The base elements contain the variables  $a$ ,  $b$ ,  $c$  and  $d$ , as defined in the first paper, plus the variables  $u$  &  $v$ , which are angles in the triangular element with the angle  $u$  next to the funnel center and  $v$  the angle at the HP center.

The domes and funnels are circular for regular layouts, such as square, equilateral triangular and regular hexagonal units. Similarly, for irregular units, as a first approximation, all domes and funnels may be assumed to be equally circular. This means the previously established equations may be used. As each base element will only occupy a part of the area surrounding a column, the following approach is used:

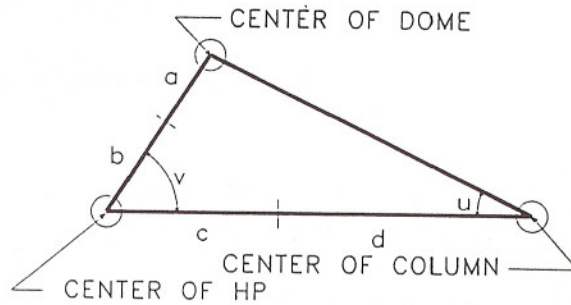


Fig. 1. SM Base Element for Free-Form Layout

Initially, the calculations assume that the columns are completely surrounded by identical fictitious base areas, each a mirror reflection of the adjacent and filling in the full 360°. This leads to the following general expressions with the number of fictitious base areas being defined as  $2m$  where  $m$  is:

$$m = \frac{180}{u} \quad (1)$$

The total contributory area for determining the fictitious column load is:

$$A_{TL} = m (a + b) (c + d) \sin v \quad (2)$$

The thrust increase in the funnel area is:

$$RN = 1.58 - 0.0247u + 0.0004 u^2 \quad (3)$$

The latter expression is derived from a curve-fitting process, using known values calculated from the regular layouts. In regular SM layouts, rectangular, triangular and hexagonal layouts, the stresses and thrusts continue unchanged from one triangular area into the next, whereas, in irregular layouts the stresses and thrusts change from one area to the next and must be analyzed as "stand-alone" units. The results are that along the borders of one unit the thrust might not exactly match those of the adjacent one. Moreover, a gradual change will take place. The solution to this problem is much the same as the one used for the free shell edges as discussed above. The unbalanced horizontal forces are carried in the shell surface to the

previously described thrust lines. Here they are balanced out by mild steel reinforcement or by tension in prestressing cables.

The actual column loads are given by the equation:

$$W_n = wA_n = \Sigma W_n / 2m_n \quad (4)$$

where  $n = 1.2 \dots n \dots$  represents the number of base elements actually located at any one column.

The total required force  $T$  to be reacted by the reinforcing along each thrust line is:

$$T = (a_1 H^D + b_1 H^{HP} + a_2 H^D + b_2 H^{HP}) / (2 \cos v/2) \quad (5)$$

Where  $H^D$  and  $H^{HP}$  are the thrusts normal to the  $a$  and  $b$  distances on either side of the same thrust line. In free-form layouts, a CAD-program may be used to determine all geometry. The final stresses are conveniently calculated from a spread sheet, containing the previously given equations.

#### 4. EXAMPLES

Initially, three regular shell layouts are shown. The first layout consists of a shell with the columns arranged in a square pattern (Fig.2); the second shows a shell supported on columns that form an equilateral triangular pattern; and the third layout utilizes columns in a hexagonal pattern (Figs. 3 and 4).

The floor plans show column-lines, thrust-lines and the resulting SM units as developed for the various column arrangements. The terms *middle-strip* and *column-strip*, as borrowed from the conventional ACI flat plate designs, are present in all three layouts. For the square layout, these entities are identical to those used for rectangular plates in the same reference code. In the developed-longitudinal sections, the perfect continuity of the various profiles of the shells are distinctly demonstrated. A quick check shows that all rules for discretizing a SM layout into part areas are complied with.

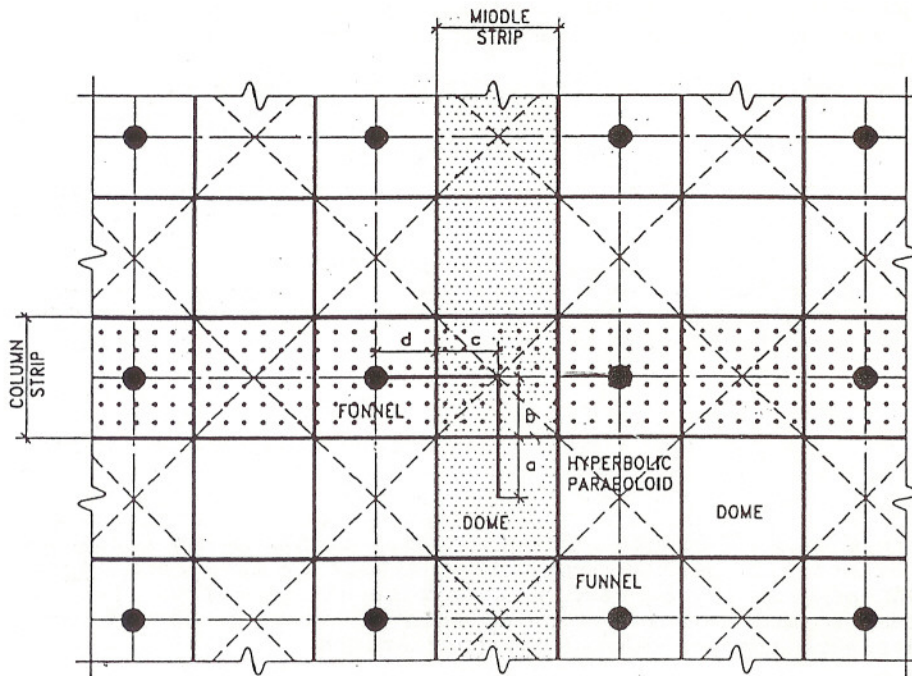


Fig. 2a. SM for Square Column Layout

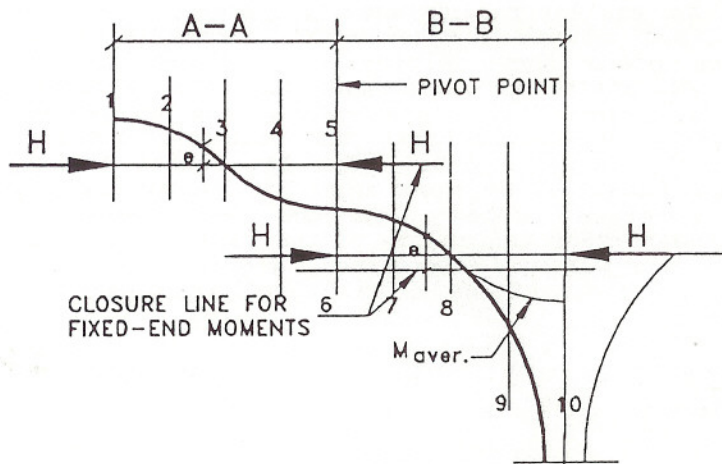


Fig. 2b. Developed Longitudinal Section for Square Layout

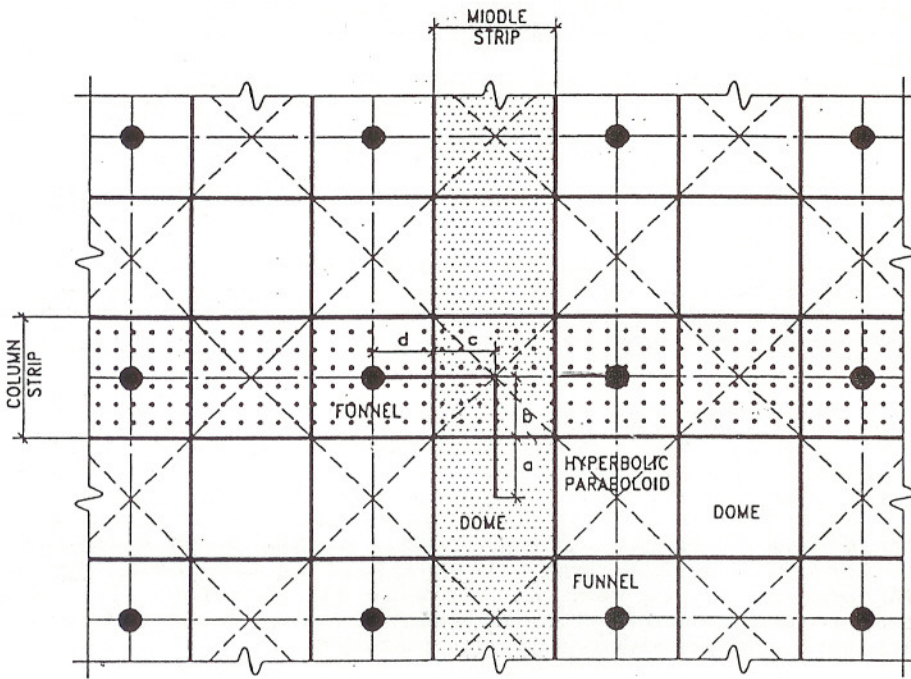


Fig. 2a. SM for Square Column Layout

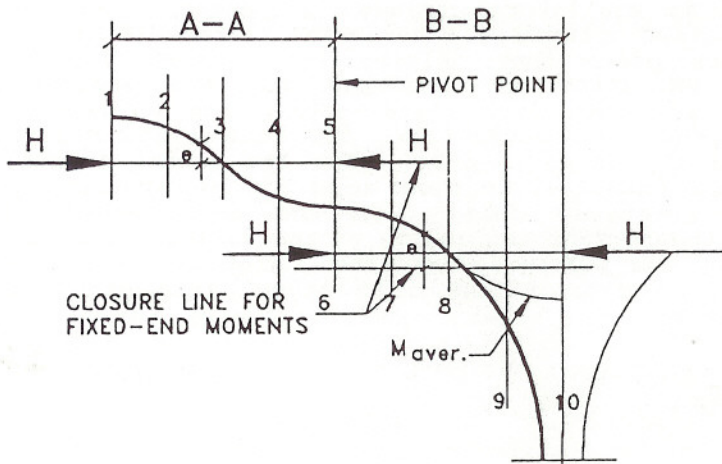


Fig. 2b. Developed Longitudinal Section for Square Layout

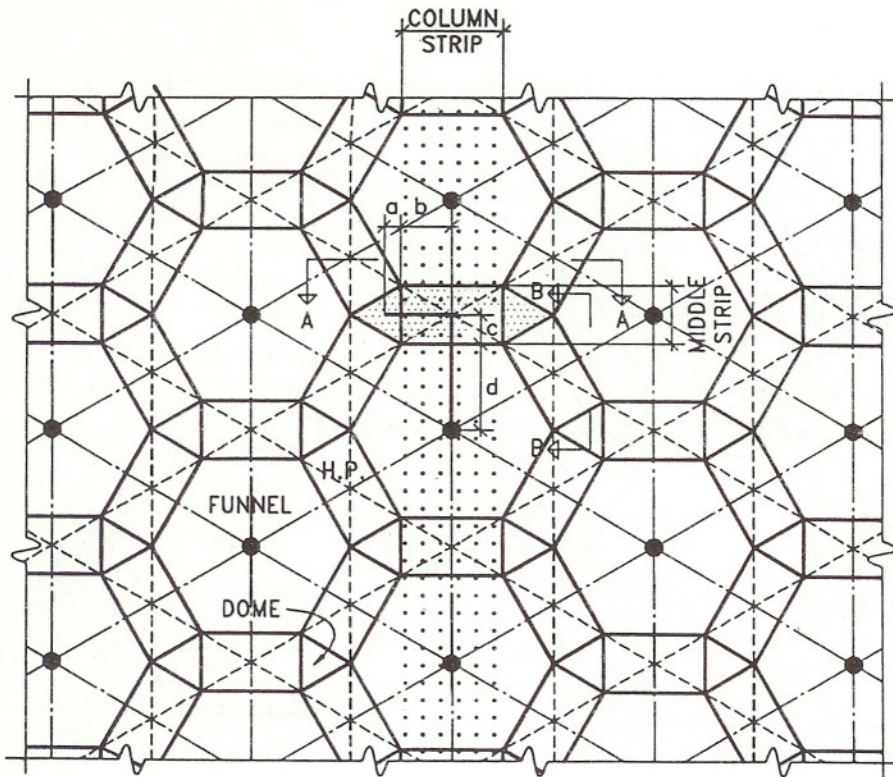


Fig. 3a. SM for Equilateral-Triangular Layout

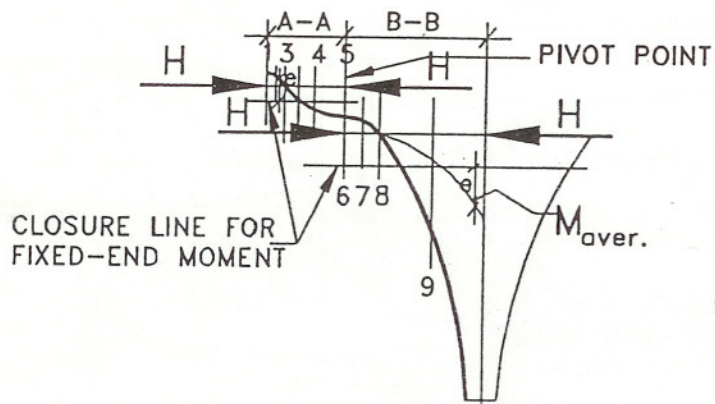


Fig. 3b. Developed Longitudinal Section for Triangular Layout

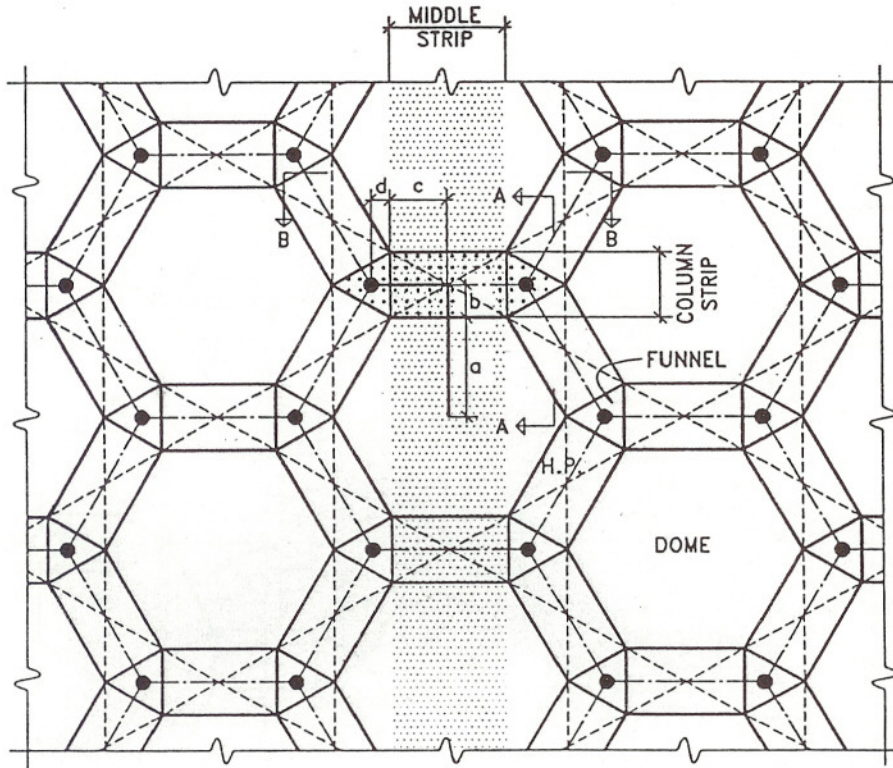


Fig. 4a. SM for Hexagonal Column Layout

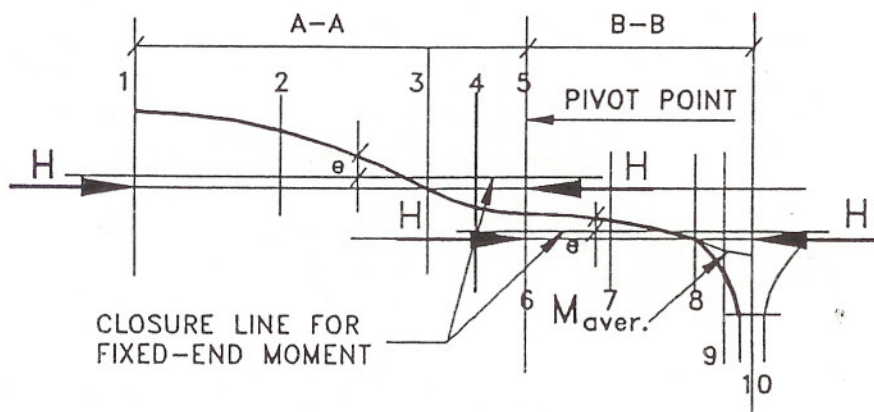


Fig. 4b. Developed Longitudinal Section for Hexagonal Layout

A simple free form layout is shown where the center column has been removed. The numbered areas comprise the domes, while the roman numerals identify the columns. On the left, the "column-strips" have been hatched and on the right, some of the base triangles are highlighted. The gradual change in stresses and thrusts, from one base triangle to the next, are shown at columns VII, VIII, IX and XII.

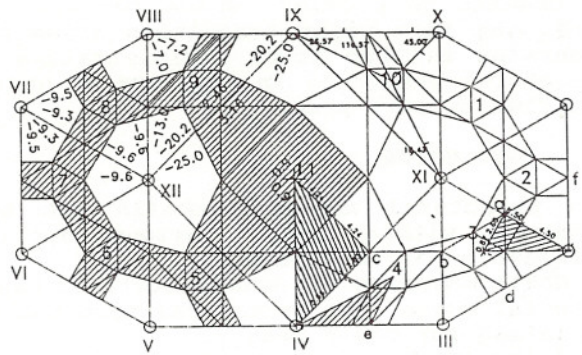


Fig. 5. FreeForm Column Layout

The next example, although not built, shows a fieldhouse, in INDIANA, USA. The program uses one composite structural membrane to place a fieldhouse, gymnasium and swimming pool under one roof. This structure, cast on a sand hill, prestressed and lifted with conventional lift-slab equipment, provides the cover in one continuous smoothly curved-concrete roof. Within the concrete surface, prestressing tendons run straight from one exterior edge, to the opposite edge. Shallow edge beams solidify the undulating, continuous edges of this 364' long structure.

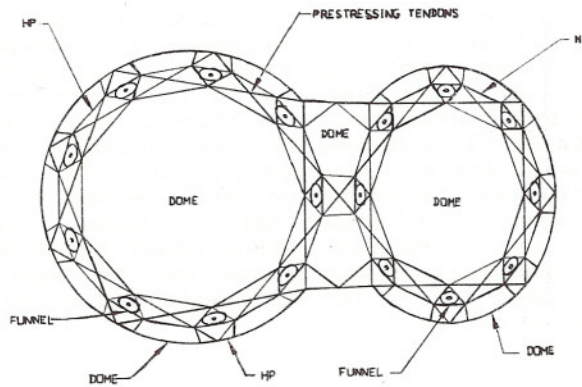


Fig. 6. Combined SM Roof for Fieldhouse, Gym and Swimming Pool

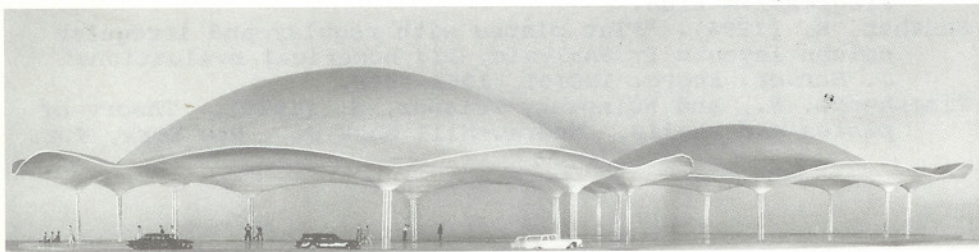


Fig. 7. Model of Fieldhouse

The 210 FT. diameter dome accommodating the fieldhouse is supported on nine columns. The eight-pointed elliptical dome is created to provide space for the olympic sized swimming pool. The five point transition areas providing space for the gymnasium and two story administrative building are irregular structural membranes in the form of unsymmetrical domes fitted together by part-areas.

##### 5. CONCLUSION

In structural membrane shells the horizontal thrusts are constant within their various domains, allowing maximum utilization of materials.

While representing true structural expressions for an optimum material usage, the structural membrane shells have been widely acclaimed for their beauty.

Adding to the economy of the structural membrane shells no roofing is expected to be required, since the entire surface, by use of prestressing, is subjected to uniform compression. This makes it free of bending moments and therefore, free of cracks.

The structural membrane shells are ideally suited for long spans. Furthermore, a blend of long and short span areas are easily accommodated. For short span structures, the structural membrane shape makes it possible to use cementitious foams with very low material strength values. Using these types of materials, water tightness and thermal insulation may be combined within the single layer of the membrane shell surface.

Several new features can be incorporated in the manufacturing of structural membranes. Long span concrete structural membranes are easily and cheaply constructed on earth mounds and lifted to their desired heights by conventional lift slab techniques. Intermediate spans may be site-cast and lifted into place with cranes. For short spans, foamed structural membrane shells sprayed on modular forms could be used. Similarly, segmented prefabricated components can be mass-produced in factories, shipped to the building site and glued or bolted together for the final roof.

##### APPENDIX I. REFERENCES

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